Crystal-size Distribution in Mixed Suspensions

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Although vacuum crystallizers are used widely, crystal-size-control methods have not been adequately analyzed. This paper supplies in part the deficiency of technical information on this problem. Industrial crystallizers may have inherent nucleation rates in excess of or below the seed-crystal requirement for the desired product-crystal size. This paper deals specifically with means of crystal-size control by removal of excess nuclei from mixed circulating suspensions as encountered in vacuum crystallizers. It is shown that under certain conditions the cumulative size distribution in the suspension varies as the fourth power of the size. Therefore, it may also be shown that the key to effective size-control procedures is the segregation time of nuclei in the fines-removal system. Procedures based mainly on crystal-size classification by hydraulic elutriation can hardly be effective unless they are also designed to meet the segregation-time requirements.

Effective correlation of the various factors which affect the performance of industrial crystallizers has been hampered by the multitude of factors involved and the obscure interrelations between them. It is for this reason that crystallization is often referred to as being more of an art than a science. The important basic relations governing crystal growth have, however, been disclosed. If these are systematically consolidated to yield derived relationships between the crystallizer operating conditions, the number of independent crystallizer variables may be reduced to simplify the correlation of the remaining factors.

The methods discussed are intended particularly for analysis of the performance of continuous vacuum crystallizers operating with mixed or semiclassified suspensions. Crystallizers of these types have been described (5). A typical single-body evaporator crystallizer, illustrated in Figure 1, is widely used, but little, if any, information has been published on crystal-size control for this type of unit. Although descriptions of the single-body types generally fail to include or depict any specific means for size control by removal of excess nuclei, segregation of fines by hydraulic elutriation in separate vessels is adaptable to this type.

The design which most strongly emphasizes means for size control by the segregation and removal of excess fines is the Oslo-Krystal type (9). Modified arrangements which have been demonstrated and described by the T.V.A. (7) are illustrated in Figure 2. The design of the two-body Oslo-Krystal type permits rather wide flexibility in the choice of crystallization conditions with a nearly completely classified suspension on one extreme and a fully mixed suspension on the other. An analysis of crystal-size distribution for classified suspensions was reported earlier by Miller and Saeman (4). More interference from salt deposits may, however, result from the relatively larger wall area in contact with solutions of appreciable supersaturation.

The use of mixed suspensions in crystallizers permits operation at higher rates than with classified suspensions though at an intermediate crystal size (7). Where the primary emphasis shifts to crystals of large size with lesser emphasis on production rate, operation with a semiclassified suspension is highly advantageous, as shown by Miller and Saeman (4). The reasons for this variation in crystallizer performance with respect to crystal size and production rate will be clarified when the principles governing size distribution are analyzed.

DERIVATION OF RELATIONS FOR CRYSTAL-SIZE DISTRIBUTION

Crystallization operations may be characterized by the fact that the inherent nucleation is either greater or smaller than the seed rate required for crystals of the specified size. This analysis deals only with cases where the nucleation rate is excessive. This situation is illustrated diagrammatically in Figure 3, where it is presumed that product crystals are removed only after they have attained full size.

The basic assumptions illustrated in Figure 3 and underlying the derivation of a relation for crystal-size distribution are

1. The suspension is completely mixed.
2. The system is operating at steady state conditions.
3. The number of crystals withdrawn as product and fines per unit time is equal to the nucleation rate.
4. The weight of crystals withdrawn as product and fines per unit time is equal to the crystallization rate.
5. Product crystals are withdrawn at full size only.

Although crystal size is usually measured as length or screen mesh, the average size and size distribution must satisfy assumptions 3 and 4 above, so that the number of crystals per unit weight is equal to the nucleation or seed rate divided by the crystallization rate.

Although the linear crystal size depends on crystal geometry, the shape coefficient is as a rule constant and so the linear size may be presumed to vary as the cube root of the volumetric size, or

$$l = k_1 v^{1/3} \qquad (1)$$

where

k_1 = shape coefficient.

For crystals growing in turbulent solution, as in a dense suspension, the rate of crystal growth is proportional to the supersaturation (1). Although the relative velocity of the solution with respect to crystals influences the crystallization rate as shown by McCabe and Stevens (3), this effect can be disregarded in this instance owing to the relatively small change in growth rate over the velocity range (0.05 to 0.15 ft./sec.) of interest for larger crystals (4). Furthermore, it is presumed that the large and small crystals are intimately mixed under conditions of hindered settling and so the differential effects of solution velocity on the growth rate of large and small crystals will be appreciably less than for suspensions of crystals segregated by size. These conditions together with the finding that each crystal axis is characterized by a unique rate of growth coefficient lead to the derivation of the principle of geometric-shape similarity for the various stages of growth experienced by crystals and to McCabe's ΔL law (2). Both large and small crystals are exposed to solution of the same supersaturation in mixed suspensions; therefore no variation in growth rate is attributable to this source. Under these conditions the linear rate of growth is constant in time and is proportional to the average supersaturation

$$\frac{dl}{dt} = k_2 s \qquad (2)$$

where

k_2 = coefficient relating supersaturation to rate of growth.

If the seed size is presumed negligible relative to the product, Equation (2) integrates to give

$$l = k_2 s t \qquad (3)$$

as the relation between the crystal size l and age t.

The number of crystals in suspension smaller than l is equal to the seed rate times the age t [Equation (3)] of crystal size l, or

$$N = St = \frac{Sl}{k_2 s} \qquad (4)$$

where N = cumulative number of crystals smaller than l.

Since Equation (4) relates the cumulative number of crystals in suspension to the size l, it may be transformed to show a similar relation between the cumulative crystal weight and the size l. The increment in crystal weight dW, associated with an increment in size dl, is equal to the product of crystal density, crystal volume, and the number of crystals dN in increment dl, or

$$dW = \frac{Dl^3 \, dN}{k_1} \qquad (5)$$

Using the derivative dN from Equation (4) in Equation (5) gives

$$dW = \frac{Dl^3 S \, dl}{k_1 k_2 s} \qquad (6)$$

which on integration ($l = 0$ being assumed for lower limit) yields

$$W = \frac{DSl^4}{4k_1 k_2 s} \qquad (7)$$

Under steady state conditions all the factors on the right-hand side of Equation (7) are presumed to remain constant except the size l of crystals in suspension.

The cumulative weight, therefore, varies as the fourth power of the crystal size. For example, crystals up to one-half product size constitute only $(\frac{1}{2})^4$, or one sixteenth, of the total weight of crystals in suspension.

The actual residence time of crystals in suspension is dependent on the weight of crystals in suspension and the production rate. For convenience, the crystal age may be expressed as a function of the draw-down time T, which is defined as the ratio of the weight of suspension W divided by the production rate P, or

$$T = \frac{W}{P} \qquad (8)$$

Since $P = DSl^3/k_1$,

$$T = \frac{W}{P} = \frac{DSl^4/4k_1 k_2 s}{DSl^3/k_1} = \frac{l}{4k_2 s} \qquad (9)$$

Substituting Equation (3) ($l = k_2 st$) in (9) yields

$$T = \frac{k_2 st}{4k_2 s} = \frac{t}{4} \qquad (10)$$

Thus it is shown that the age of product crystals for classified-product removal is four times the draw-down time. Equation (7) for crystal-size distribution is further simplified by replacing absolute size l by relative size l/L, where $L = k_2 sT$.

Thus

$$W = \frac{DSl^4}{4k_1 k_2 s} = \frac{DSTL^4}{4k_1 k_2 sT} \left(\frac{l}{L}\right)^4$$

$$= \frac{DSTL^3}{4k_1} \left(\frac{l}{L}\right)^4 \qquad (11)$$

Equation (11) is written in this form to facilitate subsequent comparisons of suspensions for classified- and mixed-product removal. The variable term $(l/L)^4$ of Equation (11) is plotted graphically in Figure 5. Since the product crystal size for classified-product removal is $4L$, the total weight of suspension is

$$W = \frac{DSTL^3}{4k_1} \left(\frac{4L}{L}\right)^4 = \frac{64 DSTL^3}{k_1} \qquad (12)$$

DERIVATION OF RELATIONS FOR CRYSTAL-SIZE DISTRIBUTION FOR MIXED PRODUCT

Withdrawal of product direct from the suspension distorts the simple seed-to-product relationship pictured in Figure 3 and necessitates the use of additional relations to derive the weight-vs.-size relationship. The new relations are depicted in Figure 4. Here it is presumed that the probability that seed crystals and crystals of intermediate size are withdrawn as product is proportional to the population density of each crystal size in the system. Since the population density of the smaller crystals is greater than that of the larger sizes, the cumulative weight-vs.-size relation will be biased so that smaller crystals account for more of the total bulk than that which corresponds to the simple fourth-power relation.

To analyze this situation one may assume that the size range is subdivided into equal size intervals where n is the population density of crystals within the interval. If no crystals were lost as product, the number of crystals entering and leaving each size interval would be the same, as it is presumed that the rate of crystal growth is independent of size. Crystals withdrawn from each interval as product will, however, change the population density, and, since the number of crystals withdrawn per unit time is proportional to the population density and the withdrawal rate,

$$\frac{dn}{dt} = -\frac{n}{T} \qquad (13)$$

where $T = V/Q = W/P$.
Integration of Equation (13) between the limits n_0 and n gives

$$n = n_0 e^{-t/T} \qquad (14)$$

where n_0 corresponds to the initial population density of seed crystals. The value of n_0 is equal to the seed rate divided by the rate of growth, or
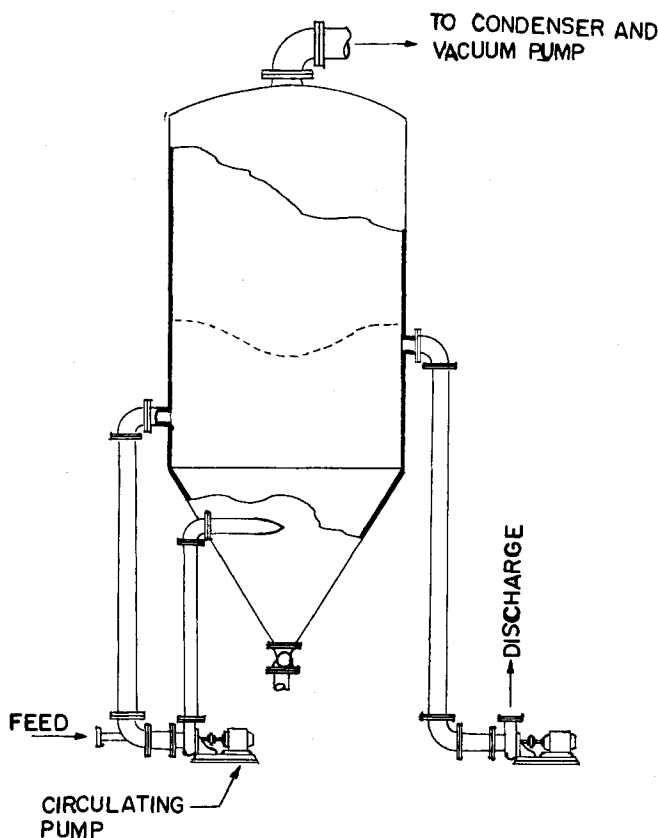
$$n_0 = \frac{S}{k_2 s} \qquad (15)$$

Fig. 1. Continuous vacuum crystallizer with pump circulation (8).

By substituting $t = l/k_2 s$ and $L = k_2 sT$, Equation (14) can be rewritten as

$$n = \frac{ST}{L} e^{-l/L} \qquad (16)$$

As shown in Equation (5), the weight of crystals in any size interval is

$$dW = \frac{Dl^3 \, dN}{k_1}$$

In this case $dN = n \, dl$, and so

$$dW = \left(\frac{DST}{k_1 L}\right) l^3 e^{-l/L} \, dl \qquad (17)$$

The integral solution for (17) is

$$W = C - \left(\frac{DSTL^3}{k_1}\right) e^{-l/L}$$
$$\cdot \left[6 + 6\left(\frac{l}{L}\right) + 3\left(\frac{l}{L}\right)^2 + \left(\frac{l}{L}\right)^3 \right] \qquad (18)$$

Since $W = 0$ when $l = 0$ the constant of integration C is $6DSTL^3/k_1$ and so

$$W = \left(\frac{DSTL^3}{k_1}\right)\left(6 - e^{-l/L}\right)$$
$$\cdot \left[6 + 6\left(\frac{l}{L}\right) + 3\left(\frac{l}{L}\right)^2 + \left(\frac{l}{L}\right)^3 \right] \qquad (19)$$

For convenience, the variable term in Equation (19) is represented as a function of the size ratio l/L,

$$f\left(\frac{l}{L}\right) = 6 - e^{-l/L}$$
$$\cdot \left[6 + 6\left(\frac{l}{L}\right) + 3\left(\frac{l}{L}\right)^2 + \left(\frac{l}{L}\right)^3 \right] \qquad (20)$$

and is plotted for reference in Figure 5. The weight of suspension up to size l is therefore

$$W = \frac{DSTL^3}{k_1} f\left(\frac{l}{L}\right) \qquad (21)$$

Approximate coincidence has been observed between the distribution function derived here and numerous samples of crystals taken from crystallizers operating with mixed-product removal from suspension. Owing to the relatively long crystal retention time, deviations may readily occur because of variation in nucleation and seed rates which induce nonequilibrium conditions in the suspension.

Equation (13) also represents the crystal-size distribution for crystals withdrawn as product. The contribution of each size interval to total production is given by

$$\frac{dP}{dl} = \frac{Dl^3}{k_1}\frac{dn}{dt} = \frac{-Dl^3 n}{k_1 T} \qquad (22)$$

Substitution of (16) in (22) converts the relation to a function of l only, and so

$$dP = \left(\frac{DS}{k_1 L}\right) l^3 e^{-l/L} \, dl \qquad (23)$$

Since Equations (23) and (17) agree in functional form, the function $f(l/L)$ plotted in Figure 5 is applicable and the cumulative product rate up to size l is

$$P = \frac{DSL^3}{k_1} f\left(\frac{l}{L}\right) \qquad (24)$$

Comparison of Equations (24) for P and (21) for W shows that $W = TP$ in agreement with the definition [Equation (8)].

The derivative curve of $f(l/L)$ (also shown in Figure 5) shows that the mixed product has a dominant size fraction for $l/L = 3$ compared to the size $l/L = 4$ for classified product. Also, half the product is greater than $3.7L$, as $f(l/L) = 3$ at $l/L = 3.7$. [$f(l/L) = 6$ corresponds to 100% of product.]

COMPARISON OF CONDITIONS FOR CLASSIFIED- AND MIXED-PRODUCT REMOVAL

Comparable operating conditions are presumed to exist in crystallizers producing classified and mixed products when the rates of crystal growth in the two systems are equal. The rate of growth is selected as a criterion since the quality of crystal structure tends to deteriorate as the rate of growth increases; so for purposes of comparison the crystals from the two systems should be of comparable quality.

The weight of the suspension carried in the system is a further criterion for comparative operating conditions since the suspension capacity of the system is fixed. The same weight of suspension will, therefore, be assumed in both cases. Since the production rates, suspensions, and rate of crystal growth are presumed equal for both systems, the draw-down time T and crystal reference size L are also the same. The two cases can therefore be equated only by an adjustment in seed rate to satisfy the equations

$$W_c = \frac{64 DS_c TL^3}{k_1} = W_m$$
$$= \frac{6 DS_m TL^3}{k_1} \qquad (25)$$

and

$$P_c = \frac{64 DS_c L^3}{k_1} = P_m$$
$$= \frac{6 DS_m L^3}{k_1} \qquad (26)$$

where the subscripts c and m refer to classified and mixed products, respectively.

In either case it is found that

$$S_c = 0.094 S_m \qquad (27)$$

For operation under comparable conditions for mixed and classified product, the required seed rate for mixed product is therefore about eleven times the number required for classified product. This difference is hardly significant on a weight basis since the extra seed crystals for mixed product are confined to sizes smaller than $2.34L$. This value is derived from Equation (16), where the factor $e^{-l/L}$ for mixed product was made equal to 0.094 to calculate the seed size for which both suspensions had equal population densities. Mixed product has 77% of its weight in crystals larger than $2.34L$. Thus 77% of the mixed product will contain the same number of crystals as is found in 100% of the classified product. The remainder of the mixed product will consist of smaller sizes.

If comparisons are to be made between equal weight percentages of mixed and classified product, it is found that the 71% fraction of the largest crystals in the mixed product contains the same number of crystals as those in 71% of the classified product. The mixed-product
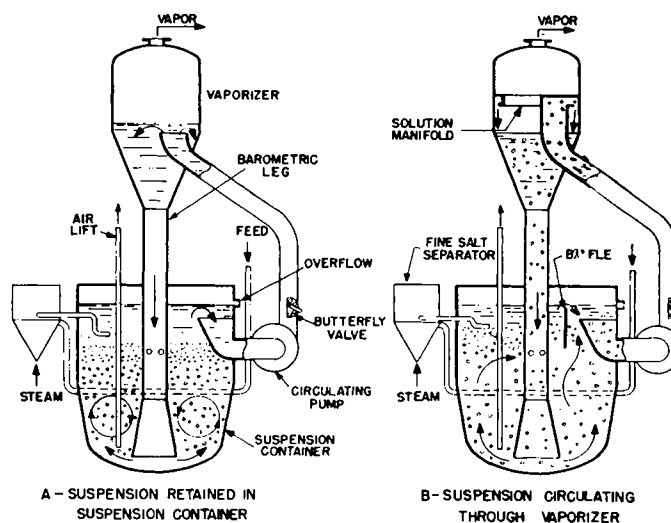

A – SUSPENSION RETAINED IN SUSPENSION CONTAINER

B – SUSPENSION CIRCULATING THROUGH VAPORIZER

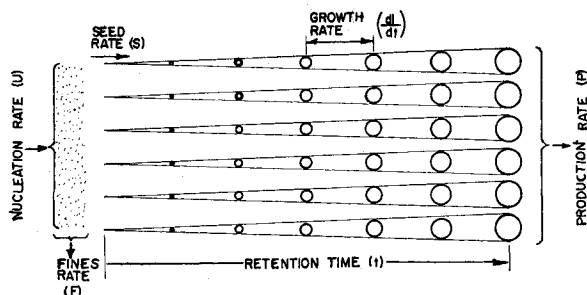Fig. 2. Two methods of operation for vacuum crystallizers for ammonium nitrate (7).

Fig. 3. Idealized crystal-size distribution for withdrawal of crystals at full size.
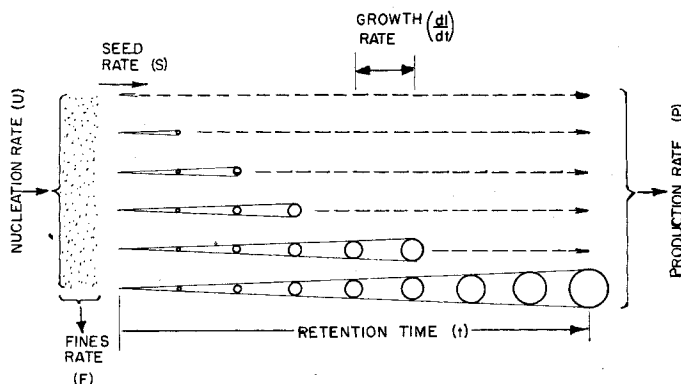

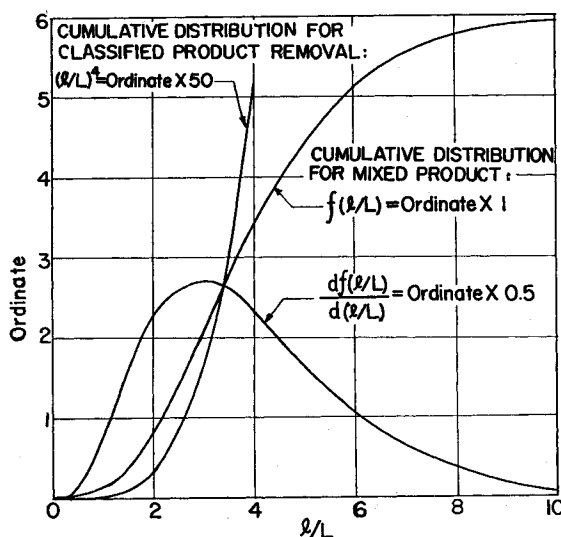**Fig. 4.** Idealized crystal-size distribution for withdrawal of crystals from mixed suspensions.

Fig. 5. Relative-distribution curves for classified- and mixed-product removal.

fraction in this case consists of crystals larger than $2.7L$. On the large-size end of the crystal spectrum it is, therefore, apparent that classified and mixed product may be regarded as equivalent for practical purposes. On the small-size end comparisons must be limited to size distribution of crystals in suspension where it was shown that the population density for mixed product must start

out about eleven times greater than the density required for classified-product removal. The cumulative weights of $0.5L$, $1L$, and $2L$ sizes in suspensions producing mixed product are, therefore, 0.3, 1.6, and 14% respectively, and the comparable percentages for suspensions on classified product are 0.025, 0.4, and 6%, respectively. In actual practice crystal-size distributions will lie between

these two limiting conditions since neither the mixed- nor classified-product removal systems will yield theoretical performance. The differences between the two systems will therefore tend to be less than those calculated above.

CRYSTAL-SIZE CONTROL

If it is assumed that the crystal geometry remains essentially constant as crystallizer conditions are varied over limited ranges, the seed rate S and reference size L must be interrelated to satisfy any given production rate. Thus as S increases, L must decrease. As L decreases, the rate of growth also decreases since the increase in total superficial crystal surface lowers the average degree of supersaturation. For constant drawdown time T the decrease in crystal size L is therefore accounted for by a slower rate of growth.

As an alternative, the increase in superficial surface associated with a decrease in crystal size may be offset by decreasing the total weight of crystals in suspension. The average supersaturation and rate of crystal growth are then unchanged and the decrease in L is accounted for by a decrease in draw-down time T. On the basis of these relations it may be inferred that for a fixed weight of crystals in suspension the maximum attainable production rate for crystals of constant quality should vary inversely as the crystal size.

SEED-RATE CONTROL

In cases where the nucleation rate is excessive, seed-rate control must be achieved by segregation and removal of excess nuclei. Since the weight of crystalline matter associated with these excess nuclei varies as the cube of the size, it is apparent that the earlier the nuclei are segregated and destroyed, the more efficient the operation.

For crystallizers operating with classified or semiclassified suspensions as illustrated in Figure 2A, excess nuclei segregate on top of the suspension below the circulating pump intake as soon as they are sufficiently large to subside. The subsided layer of fines may then be withdrawn and redissolved to destroy the excess nuclei. In this case both the excess nuclei removed from the upper layer and the product crystals removed from below are influenced by hydraulic classification so that the range of sizes is relatively narrow and both streams tend toward a similar distribution pattern. Comparisons may therefore be made directly on an l^3 basis. The ratio of fines removed to crystals produced is therefore represented as

$$\frac{N_f l_f{}^3}{N_p l_p{}^3}$$

where subscripts f and p refer to excess fines and product respectively. For

relatively large product crystals, fines may readily subside at one-tenth product size, and so a hundredfold excess of fines may readily be controlled with a fines-to-product weight ratio not exceeding 0.1.

Where fines are withdrawn from a mixed suspension into a fines trap, as shown in Figure 2B, the fines distribution is characterized more nearly by the distribution function $f(l/L)$ of Figure 5. If the product is also mixed, it will be characterized by the same distribution function and the weight of crystals in both cases will be proportional to L^3. The values of L_f and L_p, applicable to the fines and product, respectively, will be proportional to the respective draw-down times of the fines trap T_f and the product-removal system T_p. Thus, if the draw-down time of solution flowing through the fines trap is one-tenth the draw-down time of the product-removal system, it should be possible to absorb a hundredfold excess of nuclei without exceeding a fines-to-product ratio of 0.1 as in the case of classified fines and product. If, however, the fines are trapped in mixed distribution and the product is removed in classified form, the required seed rate is only one-eleventh as great, and so a hundredfold fines excess for mixed product corresponds to a thousandfold excess for classified product. The draw-down time of the fines trapping system must therefore be less than one-twentieth that of the product-removal system to keep the fines-to-product ratio under 0.1. The foregoing examples are illustrative of typical cases only. In practice numerous other combinations or variations may actually be encountered.

OPERATING DATA COVERING SIZE CONTROL AND DISTRIBUTION

The most immediate application of this relation is in the design and evaluation of means for removal of excess fines. Thus without fines removal the size of ammonium nitrate crystals grown in the T.V.A. crystallizers with mixed suspensions has been reported to be predominantly 30 to 35 mesh (Tyler standard) in size (6, 7). With fines removal the size increased to yield a major fraction of +20-mesh crystals of the type pictured in Figure 6. Operating conditions for these instances are tabulated below (6).

Screen analysis of fines removed:

Mesh size	+20	+28	+35	+48
Cumulative percentage	3	20	33	82

Screen analysis of suspension with no fines removed:

Mesh size	+16	+20	+28	+35
Cumulative percentage	0	1	38	72

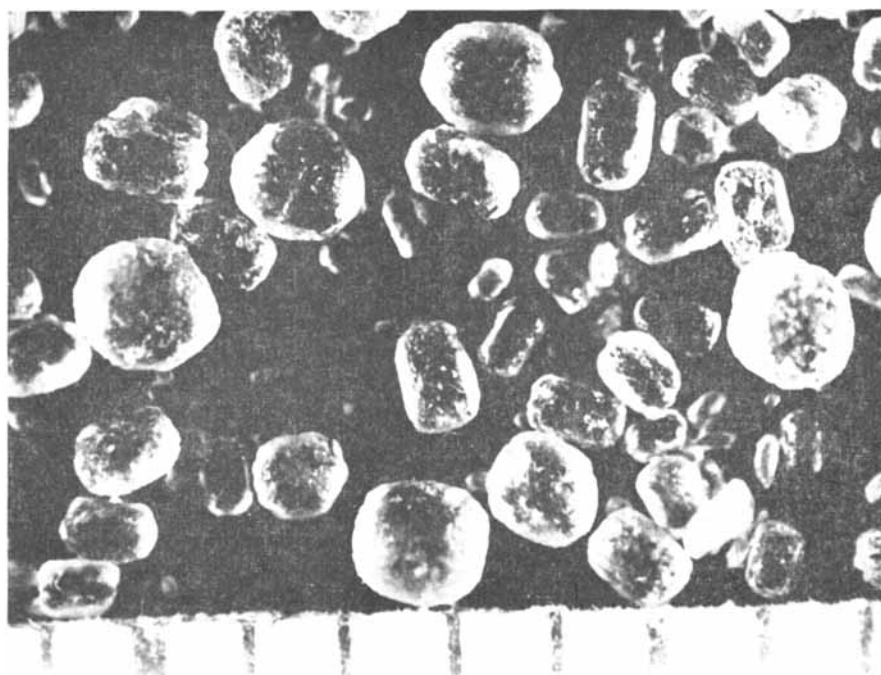Screen analysis of suspension for 15 lb./min. fines removal:

Fig. 6. Ammonium nitrate crystals produced by vacuum crystallization in T.V.A. plant (7) (millimeter scale shown along edge).

Mesh size	+16	+20	+28	+35
Cumulative percentage	12	62	89	94

The tests were made at a 140 lb./min. crystallization rate with 80,000 lb. of crystals in suspension (23,000 gal.). The suspension was circulated at 10,000 gal./min. to limit the maximum supersaturation to 0.014 lb./gal. Since the larger crystals produced in this instance are roughly twice the length of the smaller, fines removal has effected about an eightfold ($2^3 = 8$) reduction in the suspension seed rate. Inspection of the size distribution of fines removed shows the presence of substantial quantities of large crystals due to turbulent mixing between the main suspension and the trapped fines. Owing to this source of interference the full benefits of the fines trap were not realized.

The fact that 30–35-mesh (0.5 mm.) crystals were produced without fines removal serves as a definite index of the inherent nucleation rate of the system. If 2-mm. crystals were desired (fourfold size increase), the effective seed-rate requirement would be only one sixty-fourth as great. It would, therefore, be necessary to destroy sixty-three out of every sixty-four nuclei formed. To avoid loss of operating efficiency this large number would have to be isolated and redissolved at a very early age. Thus, where fines are segregated at an average size one fifth of product size, the volume of fines relative to product crystals would be about

$$63(1/5)^3 = 63/125, \text{ or } 50\%$$

If, however, the fines system is designed to segregate excess nuclei at one-twentieth the average product size the percentage would be

$$63(1/20)^3 = 63/8,000, \text{ or about } 1\%$$

Fines collection at ages one tenth to one twentieth of the average product size is within practical limits, and so size control by this means should be effective even though the inherent nucleation rate is about 100 times more than the requirements (fines removal on weight basis 1 to 10% of crystallization rate). The baffle system of fines segregation in the upper part of the T.V.A. Oslo-Krystal units (7) was reported to segregate fines from 1,000 to 2,000 gal./min. of solution (6), and the solution from the entire system (23,000 gal.) was passed through the fines segregation system on the average of 10 to 20 min. (10- to 20-min. fines draw-down time). The performance of the T.V.A. system was, however, handicapped by the asymmetric location of the baffle with respect to the currents in the suspension. Even though the fines subsided to a distinct level behind the barrier, this level fluctuated erratically and turbulence from below projected currents of larger crystals into the fines layer so that the over-all efficiency of the system was impaired. Further developments are required to improve the efficiency of this baffle design, as the fines draw-down time was one-thirtieth the product draw-down time; whereas in actual operation, with fines dissolved at 10% of the crystallization rate, the apparent fines draw-down time for an eightfold reduction in seed rate was about
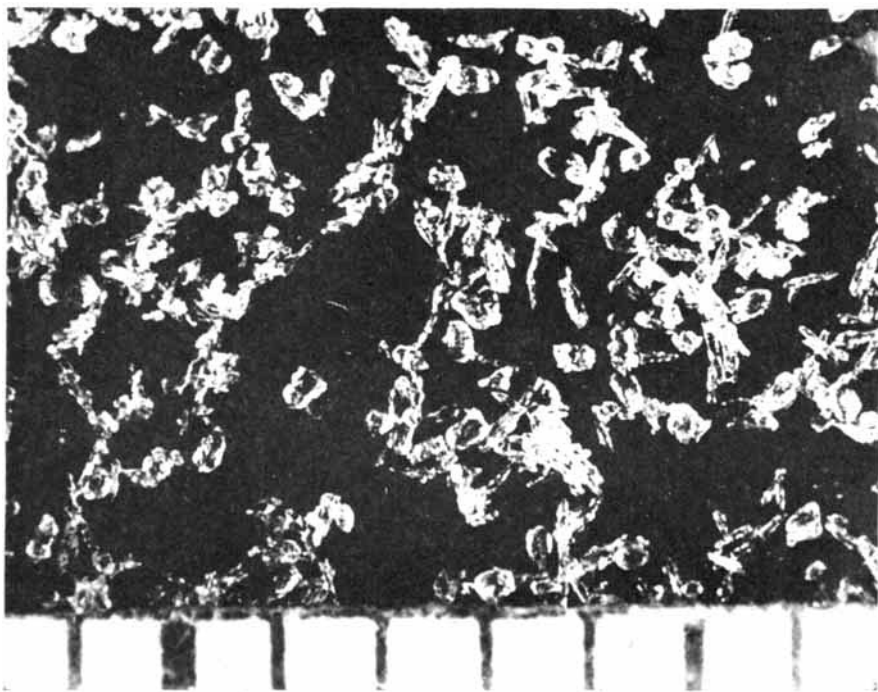
Fig. 7. Fines collected in fine-salt separator (6) (millimeter scale shown along edge).

1/4.1 of the product draw-down time $(7[1/4.1]^3 = 10\%)$. This inefficiency is corroborated by the presence of substantial amounts of coarse crystals in the photograph of the fines collected as shown in Figure 7.

The foregoing estimates of nucleation rates are only approximate, as the crystal size, density of suspension, and turbulence in the system all affect the nucleation rate, as shown by a comparison of operating conditions applicable to mixed and semiclassified suspensions (6). In the latter case predominantly 8- to 12-mesh crystals were readily grown at a 55 lb./min. rate in a 45,000-lb. suspension (18,000 gal.) by use of a fines-removal rate of less than 0.5 lb./min. A circulation rate of 7,000 gal./min. was used to permit some classification within the system, and supersaturation was limited to 0.008 lb./gal. In this case fines segregation occurred on top of the suspension below the pump intake. Since this is possibly the most effective method of fines segregation known, associated with the mildest conditions for minimizing crystal attrition, it is highly advantageous to operate in this manner whenever crystal size serves as the most important criterion of plant performance.

IMPORTANT DESIGN CRITERIA AFFECTING CRYSTAL SIZE AND SIZE DISTRIBUTION

New crystallizer applications are often approached from the viewpoint that the most effective means for product-size control lies in size-classification devices (either hydraulic or mechanical) for the crystals withdrawn from the suspension. Regardless of the type of classification device used, it is readily apparent that it cannot put out large crystals unless conditions are maintained in the suspension which are conducive to the growth of large crystals. If proper conditions for the sustained growth of large crystals are maintained in the suspension, the crystal-size-distribution functions indicate that for most applications the smaller sizes are insignificant on a weight basis, and so there is no strong need for product classification.

Positive and direct means for size control lie largely in the provision of effective means for segregation and elimination of excess nuclei. Efficient segregation methods should be effective for the elimination of at least a hundredfold excess of nuclei relative to product crystals. This implies that the nuclei must be eliminated at an average age of one-tenth the product age or less. Thus the critical-performance requirement of the segregation system is the maximum solution rate through the segregation system relative to the total capacity of the crystallizer.

Secondary requirements that must be met are the establishment of a stable fines subsistence level free of fluctuations or turbulence, so that the fines-solution system will not inadvertently overload with relatively large intermediate-size crystals.

The design of the Oslo-Krystal crystallizers fulfills several of these objectives simultaneously in that nuclei, after reaching a critical size, subside to a stable layer on top of the suspension. For operation with a classified or semiclassified suspension all the solution in the system passes through the segregation zone on each pass through the crystallizer.

For operation with mixed or circulating suspension the Oslo-Krystal units require the installation of extra baffles to permit effective fines segregation. In the latter case the ratio of solution flowing through the baffled area relative to the total circulation should be as great as possible consistent with the fact that the fines must subside to a stable level to permit efficient withdrawal of the fines. Since the basic principles governing crystal-size distribution and crystal-size control are equally applicable to Oslo-Krystal and single-body crystallizers, effective size control in the latter type can also be achieved by installation of means for the segregation and elimination of fines.

NOTATION

a = constant
D = density of crystal, lb./cu. ft.
e = base of natural logarithms
F = fines removal rate, lb./hr., no./hr.
k = constant
l = linear crystal size, ft.
L = crystal size at time T, ft.
n = number of crystals per unit size interval, no./ft.
N = cumulative number of crystals
P = production rate, lb./hr.
Q = suspension withdrawal rate, cu.ft./hr.
s = degree of supersaturation, lb./cu. ft.
S = suspension seed rate, no./hr.
t = time or crystal age, hr.
T = draw-down time, hr.
U = nucleation rate, no./hr.
v = volume of individual crystals, cu. ft.
V = volume of suspension, cu. ft.
w = weight, of individual crystals, lb.
W = cumulative weight of crystals in suspension, lb.
X = crystallization rate, lb./hr.

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